; question paper contains 7 printed pages]

Roll No. : ....

o. of Q. Paper : **607** **I**

ue Paper Code : 32357501

: of the Course : **B.Sc.(Hons.)**

Mathematics : DSE - I

of the Paper : Numerical Methods

:ster : V

: 3 Hours ***Maximum Marks : 75***

uctions :

a) Write your Roll No. on the top immediately on receipt of this question paper.

b) Use of non-programmable scientific calculator is allowed.

c) Attempt **all** questions selecting **two parts** from each question.

P.T.O.

1. (a) A scheme for approximating the squa[r…] of a positive real number a is based [o…]

recursive formula $x_{n+1} = \frac{x_{n}^{3} + \ldots}{3x_{n}^{2} \ldots}$

Construct an algorithm for approximat[i…] square root of a positive real number a [using] this formula.

(b) Show that when Newton's method is a[pplied] to the equation $\frac{1}{x} - a = 0$ the res[ulting] iteration function is $g(x) = x\,(2-ax)$. […] or otherwise, find the order of conve[rgence] of the method.

(c) Use the bisection method to determi[ne the] smallest positive root of the equ[ation] $\ln(1+x) - \cos x = 0$. Further show th[at the] theoretical error bound at each itera[tion is] satisfied.

(a) Consider the function $g(x) = 1 + x - \frac{1}{8}x^3$. Verify analytically that this function has a unique fixed point on the real line. Perform six iterations using the fixed point iteration scheme to approximate the fixed point of g(x) starting with $p_0 = 0.5$.

(b) Let g be a continuous function on the closed interval [a, b] with g : [a, b] → [a, b]. Show that g has a fixed point p in [a, b]. Furthermore, if g is differentiable on the open interval (a, b) and there exists a positive constant k < 1 such that $g'(x) \leq k < 1$ for all x belongs to (a, b), then the fixed point in [a, b] is unique.

(c) Find the approximated root of $f(x) = e$ (by the method of False Position, t $p_0 = 0$ and $p_1 = 1$ until $|p_n - p_{n-1}| < 5 \times 1$

3. (a) Using scaled partial pivoting during the step, find matrices L, U and P such tha

$$= PA \text{ where } A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}. \text{ Hence, (a}$$

the system $Ax = b$ where $b = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

(b) Use Jacobi method to solve the follo system of linear equations. Use the i approximation $x^{(0)} = 0$ and perform (iterations.

$$\begin{aligned} 4x_1 - x_2 &= 0 \\ 2x_1 + 4x_2 - x_3 &= 2 \\ -2x_2 + 4x_3 - x_4 &= -3 \\ -2x_3 + x_4 &= 1 \end{aligned}$$

Use the SOR method with $\omega = 0.7$ to solve the system of equations $Ax = b$, where

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & -6 & 3 \\ -9 & 7 & -20 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ -13 \\ 7 \end{bmatrix}.$$

Use $x^{(0)} = 0$ and perform three iterations.

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) Suppose that f is continuous and has continuous first and second order derivatives on the interval $[x_0, x_1]$. Derive the following bound on the error due to linear interpolation of $f : |f(x) - P_1(x)| \leq \frac{1}{8} h^2 \max_{x \in [x_0, x_1]} |f''(x)|$, where $h = x_1 - x_0$.

) (i) Construct the difference table for the sequence of the values

$f(x) = (0, 0, 0, \varepsilon\ 0, 0, 0)$.

(ii) Prove that :

$$\Delta(f_i g_i) = f_i \Delta g_i + g_{i+1} \Delta f_i$$

(c) Obtain the Newton's form of interp[olating] polynomial for the data set :

X	–1	0	1	2
Y	3	–1	–3	1

5. (a) Use the formula

$$f''(x_0) \approx \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

approximate the second order derivat[ive of] the function $f(x) = 1 + x + x^3$ at $x_0 = 1$, t[aking] $h = 1, 0.1, 0.01$ and 0.001.

(b) Find the highest degree of the polyn[omial] for which the for[mula]

$$f'(x_0) \approx \frac{f(x_0+h) - f(x_0-h)}{2h},$$ for the derivative provides the exact value [of the] derivative regardless of h.

(c) Derive second-order backward diffe[rence] approximation to the first order derivat[ive of] a function.

(a) Using Simpson's rule determine the approximate value of the integral $\int_0^{\pi} \sin x \, dx$. Further verify the theoretical error bound.

(b) Apply Euler's method to find the approximate solution of the given initial value problem $x' = (\sin x - e^t) / \cos x$, $(0 \leq t \leq 1)$, $x(0) = 0$, $N = 4$.

(c) Consider the initial value problem (IVP) $x' = t - x, (0 \leq t \leq 4), x(0) = 1, N = 4$ whose exact solution is given by $x(t) = 2e^{-t} + t - 1$. Obtain the solution of the IVP and compare the absolute error with theoretical error bound, assuming the Lipschitz constant L's equal to 1.

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